MUSIC AND PERCEPTION: A STUDY IN ARISTOXENUS

FAMILIAR and important though Aristoxenus is to students of Greek music, philosophers, so far as I can judge, have not always given him a fair run for his money.¹ No one would call him a great philosopher; but his arguments illuminate important aspects of the controversies of the late fourth century, and reflect light backwards onto the different views not only of music, but also of science in general, which had been held and argued over during the previous hundred years. Nor is he merely a referee in other men's contests: his ideas have a philosophical as well as a musical originality which deserves recognition. Plainly a single paper cannot hope to cover all the philosophically important aspects of his work, and I have chosen one topic which I take to be central, his conception of the relations between music and $ai\sigma\theta\eta\sigma us$.

More precisely, I shall be concerned with his views on the role of $ai\sigma\theta\eta\sigma\iotas$ in determining the nature of $\dot{a}\rho\mu\sigma\iotaa$. $\dot{a}\rho\mu\rho\iotaa$ is not the same thing as music, but is a part of it, as for instance is rhythm:² and it so happens that $\dot{a}\rho\mu\rho\iotaa$ or $\tau \partial \dot{\eta}\rho\mu\sigma\sigma\mu\dot{\epsilon}\nu\sigma\nu$ is the main subject of those passages of his work which have come down to us in completest form, under the general heading of $\dot{a}\rho\mu\rho\nu\iotaa$ $\sigma\tau\sigma\iota\chi\epsilon\hat{a}$, *Elementa Harmonica*.³ It should be understood that the term $\dot{a}\rho\mu\rho\nu\iotaa$ does not mean the same as our 'harmony'. There are various things which it can mean, particularly the tuning of an ordered scheme of intervals forming the basis for a musical scale:⁴ and here, by extension of the notion of a scale as a permissible sequence of intervals, the title $\dot{a}\rho\mu\rho\nu\iotaa\dot{c}$ $\sigma\tau\sigma\iota\chi\epsilon\hat{c}a$ is probably best understood as 'elements (or principles) of melody'—what makes this, but not that, a *tune*. For Aristoxenus, as for—say—the 'classical' composers of the eighteenth century, there are certain sequences or arrangements of notes which are melodically possible and others which are not: and in broad terms his question is what the principles are in virtue of which this is so—what is involved in the structure of a proper $\mu\epsilon\lambda\sigmas$, what is a musical sequence and what is not, and why.

It may be useful for me to explain in advance a few fairly elementary points about what the theorists discerned as the structure of the music of this period, and the terminology which Aristoxenus and his contemporaries used to discuss it. I shall need to say something about scales $(\dot{\alpha}\rho\mu\sigma\nu'\alpha\iota)$, the description of notes in scales, and about what Aristoxenus calls $\gamma\epsilon\nu\eta$.

In the Greek scales, as Aristoxenus discusses them, certain notes are 'fixed'—that is, whatever the scale, these elements of it stand in invariable intervallic relations to one another (cf. e.g. 22).⁵ For simplicity's sake (and following Aristoxenus's procedure in much of the work) I shall restrict the scope of my examples as far as possible to one segment of the scale, that extending from the note called $\mu \epsilon \sigma \eta$ —in some sense or other a basic or fundamental note⁶—down a fourth to the $\upsilon \pi \alpha \tau \eta$. $\mu \epsilon \sigma \eta$ and $\upsilon \pi \alpha \tau \eta$ are fixed, always a fourth apart. Between them lie two notes, $\pi \alpha \rho \upsilon \pi \alpha \tau \eta$ and $\lambda \iota \chi \alpha \nu \delta s$; and these are not fixed. Though any scale, going down from $\mu \epsilon \sigma \eta$, goes $\mu \epsilon \sigma \eta$, $\lambda \iota \chi \alpha \nu \delta s$, $\pi \alpha \rho \upsilon \pi \alpha \tau \eta$, $\upsilon \pi \alpha \tau \eta$, and covers a fourth in doing so, the intervals between the notes within the tetrachord are variable, and with certain systematic kinds of variation in these intervals we get what Aristoxenus calls change of $\gamma \epsilon \nu \sigma s$. There are three such $\gamma \epsilon \nu \eta$, diatonic, chromatic, enharmonic (cf. e.g. 44), and through appropriate changes in the relevant intervals, the tetrachord can be converted into a segment of a scale in any of them. It is also possible, as we shall see, to admit

¹ Historians of philosophy have tended to see him primarily as a source of information about other philosophers, particularly Pythagoreans. To take a more or less random sample of the standard authors, Robin barely mentions him, Zeller gives him a few pages, couched in very general terms, and Gomperz ignores him altogether.

² Cf. Plato Rep. 398d 1-2, Aristox. El. Harm. 1, Ps.Plutarch de Mus. 1142f.

³ The three books of the *Elementa Harmonica* as we now have them do not form a single unified work. For an account of opinions and arguments concerning their nature and relationships, see R. da Rios, *Aristoxeni Elementa Harmonica* (Rome 1954) *Prolegomena* IV, cvii– cxvii. ⁴ Originally the tuning of the strings of the lyre: cf. Heraclitus fr. 51: derivatively, the special varieties of tuning which form different classes of scale, including those associated with the names of the so-called 'modes' in Rep. 398-400. Cf. Ar. Pol. 1276b8 and elsewhere. See also e.g. I. Henderson, 'Ancient Greek Music' in the New Oxford History of Music (Oxford 1957) i 347-9, 384 ff.

⁵ Numbers in brackets in the body of the text refer to sections of Aristoxenus *El. Harm.* The two most useful editions are by H. S. Macran (Oxford 1902) and R. da Rios, cited at n. 3 above.

⁶ See for instance Aristotle's rather obscure remark at *Met.* 1018b29. Other useful passages may be found cited s.v. in LSJ.

minor variations on these intervals within a $\gamma \epsilon \nu os$ —e.g. raising or lowering the relative pitch of the $\lambda \iota \chi a \nu \delta s$ by a very small amount—thus giving what are called different 'shades', $\chi \rho \delta a \iota$, of the $\gamma \epsilon \nu os$ (24–25, 49). These possibilities are central to my argument, as are the senses in which a note, moved up or down to function in a scale of a different $\gamma \epsilon \nu os$ or $\chi \rho \delta a$, remains despite its new pitch the same note ($\phi \theta \delta \gamma \gamma os$).

The questions I want to discuss take us beyond pure musicology and into philosophy. Aristoxenus is not simply investigating agreed phenomena in standard ways: he is expressing, and vigorously arguing for, a particular conception of what music is, and in what the science of the study of music properly consists. He shows marked antagonism to views which differ from his own, notably those of the Pythagoreans and some persons whom he called the $\dot{a}\rho\mu\sigma\nu\kappa\sigma i$, and he is equally willing to enter disputes with both Plato and Aristotle. He in fact finds no one to agree with, and is jealous of his picture of himself as an innovator.

Aristoxenus' account of the relation between music and $ai\sigma\theta\eta\sigma\iotas$ is at the heart of his general position. I shall argue that the new understanding of the nature of music and its principles to which it leads—for I think that it is an innovation—is fruitful and attractive; but also that it generates difficulties from which I am not sure that he can disentangle himself.

Aristoxenus refers frequently and with emphasis to music as an $al\sigma\theta\eta\tau\delta\nu$. We shall best find out what this is supposed to mean by looking at his attacks on those of his rivals and predecessors who are represented as somehow denying it.

In section 32 he poses as his general question $\pi\epsilon\rho i \mu\epsilon \lambda ous \pi a \nu \tau \delta s$, $\pi \omega s \pi o \tau \epsilon \pi \epsilon \phi \nu \kappa \epsilon \nu \dot{\eta} \phi \omega \eta \dot{\epsilon} \pi i \tau \epsilon i \nu o \mu \epsilon \nu \eta$, $\kappa a i \dot{a} \nu \epsilon \mu \epsilon \nu \eta$, $\tau i \theta \dot{\epsilon} \nu a i \tau \dot{a} \delta i a \sigma \tau \dot{\eta} \mu a \tau a$: that is, in what natural or proper order of intervals a melody can move upwards and downwards. The ordering of this movement is a matter of natural law ($\phi \nu \sigma i \kappa \eta \kappa i \nu \eta \sigma i s$), and is not merely random; and in his account of it he will try to offer $\dot{a} \pi o \delta \epsilon i \xi \epsilon i s \dot{\delta} \mu o \lambda o \gamma o \nu \mu \epsilon \nu a s$, $\tau o i s \phi a i \nu o \mu \epsilon \nu o s$. I shall say more later about what this means. For the present let us concentrate on his contention that in this respect he differs from his predecessors. Some of them, he says, $\dot{a} \lambda \partial \sigma \rho i \rho \partial \sigma v \partial \nu \nu \tau \epsilon s$ —that is, introducing extraneous or irrelevant reasoning—and rejecting $a i \sigma \theta \eta \sigma i s$ as inaccurate, invented 'rational' principles ($\nu o \eta \tau \dot{a} s a i \tau i a s$) and asserted that height and depth of pitch consist in $\lambda \delta \gamma o v s \tau i \nu \dot{a} s d \rho i \theta \mu \hat{\omega} \nu$ and $\tau \dot{a} \chi \eta \pi \rho \dot{o} s \ddot{a} \lambda \eta \lambda a$, relative 'speeds'.⁷ In doing this, Aristoxenus complains, they are $a \lambda \lambda \sigma \tau \mu \omega \tau \dot{a} \tau \omega \delta \dot{o} \rho v v \dot{a} s \dot{a} \rho v \sigma v \dot{a} s$.

We have two accusations, then: that of introducing extraneous reasoning or irrelevant conceptions, and that of making assertions contrary to the $\phi a i v \delta \mu \epsilon v a$, the 'appearances', whatever exactly it is that Aristoxenus wishes to indicate by this term.

As regards the first of these, it is pretty clear in rough outline what he means, though we shall be able to fill it in more precisely as we go along. Music is something which we hear. Height and depth of pitch are perceived qualities of sound, and need to be investigated as such. They are not rates of vibration, or of any other kind of physical movement, and they are not numerical ratios. Here it is worth briefly focussing on another passage (8–9), where he is trying in a preliminary way to mark off musical sounds from others. Non-musical sound, and in particular speech, moves up and down in pitch $\sigma uv \epsilon \chi \hat{\omega} s$, continuously; whereas musical sound moves by intervals, remaining stationary at the points of arrival between leaps. Now this account, he says, is to be taken $\kappa a \tau a \tau \eta v \tau \eta s a i \sigma \theta \eta \sigma \epsilon \omega s \phi a v \tau a \sigma i a v$. The question whether in physical fact the voice can be said to move, $\kappa uv \epsilon i \sigma \theta a \iota$, across the range of unsung pitches within the interval, and then to come to a standstill, $i \sigma \tau a \sigma \theta a \iota$, at a given $\tau a \sigma \iota s$ (pitch), is nothing to do with the present enquiry: $\epsilon \tau \epsilon \rho a s$ $\epsilon \sigma \tau \iota \sigma \kappa \epsilon \psi \epsilon \omega s \kappa a \iota \pi \rho \delta s \tau \eta \nu \epsilon \psi \epsilon \sigma \tau \omega \sigma a v \pi \rho a \gamma \mu a \tau \epsilon i a v$. The another in the interval, and then to come to a standstill, $i \sigma \tau a \sigma \theta a \iota$, at a given $\tau a \sigma \iota s$ (pitch), is nothing to do with the present enquiry: $\epsilon \tau \epsilon \rho a s$ $\epsilon \sigma \tau \iota \sigma \kappa \epsilon \psi \epsilon \omega s \kappa a \iota \pi \rho \delta s \tau \eta \nu \epsilon \nu \epsilon \sigma \tau \omega \sigma a v m \rho a \gamma \mu a \tau \epsilon i a v$. The answer to that kind of question, it makes no difference: the proper criterion here is that one kind of sound is *perceived as* continuously shifting in pitch, the other as moving to and from stationary points by intervals.

⁷ The association of pitches with 'speeds', as contrasted with lengths (primarily of strings) seems to originate with Archytas, who appears to have linked them with the speed of a sound's propagation (DK 47, B1, A19a). This theory is adopted at least sometimes both by Plato (*Tim.* 80a-b) and Aristotle (e.g. *de Gen. An.* 786b7 ff.): it seems also to be one of the theories criticised by Theophrastus in his attack on the number-theorists (see Porphyry's *Com*-

mentary on Ptolemy's Harmonics [ed. Düring] 61.22-65.15, especially 63.19 ff.). Their connection with speeds of vibration is apparently due to Heracleides (reported in Porphyry *op. cit.* 29.27-31.21). On the whole subject, the most useful discussion still seems to be that in K. von Jan, *Musici scriptores Graeci* (Leipzig 1895) i 134-41.

Aristoxenus is plainly not arguing here that physical theories of sound-production in general, or any particular theories, are false. He is claiming only that they have nothing to do with the study of music. However it may be caused, the musical just *is* what is perceived in one way, the non-musical what is perceived in another.

The accusation of 'extraneous reasoning' seems in part to refer to the attempt to define musical relations in terms of mathematical ones—whether between entirely abstract quantities, between lengths of vibrating strings, between rates of vibration, or whatever. This the Pythagoreans, followed by Plato and in part by Aristotle, had notoriously tried to do, and it is, according to Aristoxenus, entirely misguided. To define, for example, the octave as the ratio 2:1 is the merest nonsense: the octave is just what we hear as a certain concord, and it is that independently of any mathematical analyses which may be applied to the conditions of its production.

The quarrel is to a great extent about the aim of musical analysis-in what terms something obscure is to be explained in order for it to count as 'explained'. As Macran noted in his edition,⁸ the point is well made by a contrast between the Aristoxenean and the Pythagorean definitions of a tone. A tone (τόνος) is not something immediately 'given': it does not come to the notice of our senses already neatly labelled with its name. It needs so specifying as to be readily identified in terms of things which are given or understood; and whereas the Pythagoreans9 define it as the difference between two sounds whose vibration-rates (or otherwise specified $\tau \dot{\alpha} \chi \eta \pi \rho \dot{\delta} s \ddot{\alpha} \lambda \eta \lambda a$: see n. 7) stand in the ratio 9:8, Aristoxenus (21) defines it as the difference between the intervals of a fourth and a fifth. In fact Macran's remarks need supplementing, since the Pythagoreans also use what is verbally the same formula as Aristoxenus's.¹⁰ But for them the expressions 'fourth' and 'fifth' refer to the intervals between two notes whose $\tau \dot{\alpha} \chi \eta$ stand in the ratios 4:3 and 3:2 respectively, and the size of the tone follows as an inference,¹¹ whereas for Aristoxenus the fourth and the fifth are simply certain heard concords, and nothing can be inferred from the formula about the mathematical value of the tone. Why this account seems adequate and appropriate to Aristoxenus will emerge more fully later, but crudely it is because the fourth and the fifth are intervals which the ear can accurately identify, and it is possible, as we shall see, to construct a tone through operations involving accurately perceivable concords alone.

Aristoxenus's other charge against these theorists is that what they say is contrary to the $\phi a_{i\nu} \delta_{\mu} \epsilon_{\nu} a$, and it is a good deal less obvious what he means by that. It is perfectly true that there *are* $\phi a_{i\nu} \delta_{\mu} \epsilon_{\nu} a$, facts of experience ascertainable by ear, which the Pythagorean system cannot readily accommodate. As Lippman says, 'Tones can be divided into halves, the fourth consists of $2\frac{1}{2}$ tones, the cycle of twelve fifths returns to the original pitch: all impossible notions from the Pythagorean point of view, but easy to demonstrate in Aristoxenean harmonics.'¹² Unfortunately, though Aristoxenus does discuss two of these $\phi a_{i\nu} \delta_{\mu} \epsilon_{\nu} a$, he nowhere argues, as admittedly he might have done, that number-ratio theories cannot accommodate them. What he does say on the subject is actually quite different, and very interesting indeed.

In sections 46-50 he sets out to explain the differences between the $\gamma \epsilon \nu \eta$; and this leads him into a sustained attack on certain mathematical conceptions of the nature of, and the relations between notes. In different $\gamma \epsilon \nu \eta$, as I have explained, the notes intermediate between the fixed points $\mu \epsilon \sigma \eta$ and $\nu \pi \alpha \tau \eta$ vary in position. Aristoxenus here argues that the $\lambda \iota \chi a \nu \delta s$ can move over the range of a tone, and the $\pi a \rho \upsilon \pi \alpha \tau \eta$ over that of the smallest diesis, i.e. a quarter-tone (46-47). And, he goes on (47), some people are astonished ($\theta a \upsilon \mu \alpha \delta \upsilon \sigma \iota$) that we continue to call this note the $\lambda \iota \chi a \nu \delta s$ when its intervallic relation to the fixed notes changes. That of $\mu \epsilon \sigma \eta$ to $\nu \pi \alpha \tau \eta$ is invariant: this relation is what makes them $\nu \pi \alpha \tau \eta$ and $\mu \epsilon \sigma \eta$. Hence we must surely allow that notes standing at different intervals from the $\mu \epsilon \sigma \eta$ are different notes ($\phi \theta \delta \gamma \gamma \upsilon \iota$), and not the same one. In general, notes bounding unequal intervals should be different notes, and notes bounding equal intervals should be the same notes. The background assumption of this position is plainly that—once we have taken some note or other as our starting point—other notes are to be defined in relation to it strictly by reference to the interval which they form with it. Aristoxenus is

⁸ Macran 245.

⁹ E.g. DK 47 A16, A17.

¹⁰ E.g. Euclid, Sect. Can. 13.

¹¹ Loc. cit.

¹² E. A. Lippman, *Musical Thought in Ancient Greece* (New York and London 1964) 150.

arguing against any such criterion for the identity of a note, whether the thing is done by Pythagorean ratios or not.¹³

He has a variety of answers. To begin with, the adoption of the rule 'same interval, same bounding notes' would be a remarkable innovation ($\mu \acute{e}\gamma a \tau \iota \kappa \iota \nu \epsilon \acute{\iota} \nu \acute{e}\sigma \tau \iota \nu$), since there are many pairs of notes distinguished in the ordinary nomenclature whose members stand at the same distances apart. They differ, not in their intervallic relation to all other notes, but by what he calls their $\delta \acute{v} \alpha \mu \iota s$, their function, a conception to which I shall return shortly.

The converse requirement, that each distinct interval from a given note must designate a distinct $\phi \theta \delta \gamma \gamma \sigma s$, would demand an infinite number of $\phi \theta \delta \gamma \gamma \sigma s$ and an infinite vocabulary (48). For mathematically there is no limit to the number of locations within its total range which the note we call $\lambda \iota \chi a \nu \delta s$ might occupy, and on this theory each locus will mark a different note. And musically there is no reason to restrict the number of possible loci within that range, let alone to restrict it to one locus: there is no musical requirement on us to prefer one 'shade' of a $\gamma \epsilon \nu \sigma s$ to another—i.e., to insist on this as opposed to that minor variation of tuning. If, perhaps, such special loci might be picked out by specifically mathematical criteria—that this interval and not that can be expressed as a ratio between integers, for instance—there is no reason at all why such considerations should place any constraints on music.

Here we come to the central point. Given a particular position of the $\lambda_{i}\chi av \delta s$, the ear will hear a scale of the appropriate $\gamma \epsilon v \sigma s$. Given a position only marginally higher or lower, the ear may indeed detect a difference, but it will still recognise the same scale, differently coloured or 'shaded'. If we insist on mathematical equalities and inequalities as our sole criterion we shall, Aristoxenus says (48), be abandoning $\tau \eta v \tau \sigma v \delta \mu o lov \tau \epsilon \kappa a d d \sigma \mu o lov \delta d \delta \eta v \omega \sigma u v$. Perceived similarities simply do not correspond to mathematical ones, and it is the *perceived* similarities which constitute properly musical groupings or categories. For instance, there is the term $\pi v \kappa v \delta v$, literally 'compressed', which is used to refer to pairs of small intervals: their common feature is that when heard together they make a compressed, crunchy sort of sound ($\pi v \kappa v \delta v \tau u v s \phi \omega v \eta$). If we are compelled to limit the use of the term $\pi v \kappa v \delta v$ to a single mathematical relation, we shall have no means of referring to what is actually *there*, as *heard*, a feature common to a whole collection of intervals lying within a range whose limits can be determined by $a i \sigma \theta \eta \sigma i s$ alone. $\dot{\epsilon} \mu \phi a i v \epsilon a recognises one series of notes as the same scale as another, it is the same, and its notes$ are the same, despite their mathematical divergences (48–9).

This explains, I think, the principal sense in which treating musical relationships as being fundamentally mathematical ones leads to conclusions contrary to the $\phi a_{i\nu} \phi_{\mu} \epsilon_{\nu} a_{.}$ We shall see that Aristoxenus is not by any means claiming that mathematics has no part to play in musical analysis: what he is insisting is that the mathematical tools must be applied to things recognisable as *heard*, and further, as I shall try to explain below, that the mathematical relations employed must themselves be specifiable as, or reducible to, relations identifiable by $a_{i\sigma}^{i\sigma} \eta_{\sigma is}$.

Before I turn to these points, I should add a word or two more about the principles governing the identity of notes. There are two senses in which Aristoxenus is insisting that a note remains the *same* note irrespective of mathematically specifiable shifts. First, a note remains e.g. the $\lambda \iota \chi a \nu \delta s$ of an enharmonic scale, despite minor variations of pitch, just so long as the ear recognises the scale as enharmonic, and the note as that next below the $\mu \epsilon \sigma \eta$ (49). Secondly, a note remains $\lambda \iota \chi a \nu \delta s$ over a much wider range of variation, right through the $\gamma \epsilon \nu \eta$, just so long as the ear recognises it as being that note which by nature, $\phi \iota \sigma \epsilon \iota$, stands in that position on the scale. It is said to be the same note by having the same function, $\delta \iota \nu a \mu \iota s$ (49).

Concerning this notion of function we evidently need to enquire by what means we apprehend something as 'having the same $\delta i \nu a \mu i s$ '. A certain amount is plain enough: in particular, that while hearing a note as being of a given pitch requires only that we hear that note, hearing it as performing a given function requires its relation to a musical context and its location

¹³ None of the theorists whose work we know seems to have adopted a view quite as crude as that which Aristoxenus here criticises. The Pythagoreans, despite their devotion to mathematics, were well aware of the distinctions he is making, as is shown by Archytas's work on the three $\gamma \epsilon \nu \eta$ (DK 47 A17). But Aristoxenus wishes to emphasise his concept of $\delta \nu \mu \mu s$, in particular its nonmathematical basis: and he would not be the first or the last polemicist to enhance his argument by erecting straw opponents for speedy demolition. within a structure. Quite a close parallel can be made with modern expressions such as 'leading note'. If someone sings up the first seven notes of a major scale we can 'hear' that the note he has arrived at is the leading note of that scale: if without context he merely sings a note, we can hear only its pitch, and cannot assign it any function.

So far Aristoxenus's conceptions seem to parallel ours quite satisfactorily. Beyond this, unfortunately, it is a matter which he leaves disappointingly vague. But certain of his observations may be helpful. At 33, in a more or less methodological passage, he tells us that the whole of our musical analysis must be based on the judgments of ἀκοή and διανοία, and whereas it is the task of άκοή to judge the size of intervals, it is by διανοία that we θεωροῦμεν τὰς τούτων δυνάμεις. It is a pity that although editors and commentators have made much of this remark,¹⁴ he does not himself follow it up. He is swept instead into a further discussion of the central role of sense-perception, implicitly continuing his contrast of correct procedure with that of the Pythagoreans, whose claims, like those of the geometers, are independent of the evidence and accurate training of $ai\sigma\theta\eta\sigma s$, and hence do not count as referring to music at all. In a later passage (38-9) he again refers to ἀκοή and διανοία as judges of musical distinctions; but here he passes at once to the claim that understanding of music is compounded of allograus and $\mu\nu\eta\mu\eta$. Speculatively, we might reconstruct his position as being that perception identifies intervals, and memory stores their sequence, thus creating the material for the sort of 'context' mentioned above; while the role of δ_{iavoia} is to identify the sequences not merely as sequences of intervals, which would be musically meaningless, but as forming or implying structures within which the notes stand in functional relationships to each other. Beyond this we cannot say how the analysis might have continued. It is plain only that $\dot{\eta} \tau o \hat{\nu} \mu \epsilon \lambda o \nu s \phi \dot{\nu} \sigma \iota s$ is not to be specified in terms of intervallic relations alone, but also and primarily by reference to musical $\delta \nu \nu \dot{a} \mu \epsilon \nu s$, functions. (See also his passage on notation, 39-40.)

We can gather rather more about the status of the $\gamma \epsilon \nu \eta$ and their relation to $a i \sigma \theta \eta \sigma \iota s$. Ultimately the distinctions between them are to be made in terms of differences in perceived character. We can see this, for instance, in Aristoxenus's complaints about those modern musicians who invariably restrict their $\lambda \iota \chi a \nu o \iota$ to the higher positions—in or near the diatonic $\gamma \epsilon \nu \sigma s$: $\tau o \iota \tau \sigma \delta$ $a i \tau \iota o \nu \tau \sigma \delta \delta c \sigma \theta a \iota \gamma \lambda \nu \kappa a \iota \nu \epsilon \iota$, he says; and if they try to play enharmonic they inevitably shift towards the chromatic, $\sigma \nu \nu \epsilon \pi \iota \sigma \pi \omega \mu \epsilon \nu o \nu \tau \sigma \iota \mu \epsilon \lambda o \nu s$, destroying the character of the melody (23).

But the distinctions between $\gamma \epsilon \nu \eta$ are subtle and not obvious. We need to use not just the ear, but the trained ear, to discover their various $\phi \nu \sigma \epsilon \iota s$. Aristoxenus reverts many times to this theme (e.g. 22-3, 32-3, 34-5, 40-1), and invariably treats the $\gamma \epsilon \nu \eta$ not as invented, but as discovered, and as present already in the nature of music for the student to grasp. In one passage he lists them in the order in which $\dot{\eta}$ $\tau o \nu$ $\dot{a} \nu \theta \rho \omega \pi o \nu$ $\phi \nu \sigma \iota s$ comes across them, and remarks that it is only $\mu \delta \lambda \iota s \mu \epsilon \tau \dot{a}$ $\pi \delta \lambda \delta \nu$ $\pi \delta \nu \sigma \upsilon$ that $a \ddot{\iota} \sigma \theta \eta \sigma \iota s$ becomes accustomed ($\sigma \nu \nu \epsilon \theta \dot{\iota} \zeta \epsilon \tau a \iota$) to the enharmonic (19, *cf.* also 47-50 and 52).

Given this conception, it becomes far from obvious why he believes that there can be no other $\gamma \epsilon \nu \eta$ (44). His procedure is, in the main, to ask what features can be found to link (a) all musical sequences recognisable as melodious, and (b) all such sequences recognisable as having a certain fundamental character. He finds, among many other things, the common $\delta u\nu \dot{a}\mu\epsilon \iota s$ in answer to the first question, and the directly perceived but analysable character of the $\gamma \epsilon \nu \eta$ in answer to the second. But it is plain, even explicit in one passage (35), that he is considering only existing melody: his subject matter is what we do recognise as musical: and because his method is at least in intention rigorously empirical, and because the principles ($\dot{a}\rho\chi a\hat{\iota}$) which he derives are constructed precisely to cover those cases which are recognised as musical and to rule out all others, it is perhaps not surprising that the possibility of extrapolating to admit wholly new kinds of musical sequence escapes him.

Seductive though this kind of criticism is, it is also pretty woolly, and makes no serious dents in Aristoxenus's procedure or his results. There is, however, a much more crucial and much more precise theoretical difficulty in his acceptance and analysis of the existing $\gamma \epsilon \nu \eta$. I should like to approach it rather gradually, setting out one or two other central theses on the way.

¹⁴ Cf. e.g. Lippman 149-50.

In a number of cantankerous and rather difficult passages (2, 7-8, 27-8, 38, 53), Aristoxenus has harsh words to say about his opponents the $\delta\rho\mu\sigma\nu\kappa\sigma i$ for their adoption of a procedure which he calls $\kappa\alpha\tau\alpha\pi i\kappa\nu\omega\sigma\iota s$, compression. It arises in connection with the attempt to express in diagrammatic form the relations between the various modes in a single structure. A mode, for these purposes, is an ordered sequence of intervals: any mode, Dorian, Phrygian, or whatever, contains the same intervals but in its own peculiar sequence, and each may appear in all the three generic forms, enharmonic, chromatic, diatonic. If we take the enharmonic versions, which involve quarter-tones, it is possible so to choose the pitch-relations between the various modes that the range of the notes used in the expression of them is as small as it can be, a sequence of 28 consecutive quarter-tones, or enharmonic 'dieses'. Onto this sequence all the modes in their enharmonic form can be mapped, the $\mu\epsilon\sigma\eta$ of each standing at a distance of 3 dieses from its predecessor. This is what Aristoxenus means by $\kappa\alpha\tau\alpha\pii\kappa\nu\omega\sigma\iota s$.

We can gather from what he says in the sections I have mentioned that the purpose of representing the various modes in set intervalic relations to one another is to explain the possibilities of modulation between them ($\mu\epsilon\tau\alpha\betao\lambda\eta\sigma\nu\sigma\tau\eta\mu\alpha\tau\kappa\eta$). Now the general principle of intermodulation in the later Greek theorists is such that it is possible only if the mode from which you start and that to which you move have in common not only particular pitches, but pitches which are, as Bacchius puts it, $\delta\mu\rhoioi\kappa\kappa\alpha\tau\dot{\alpha}\tau\dot{\eta}\nu\tauo\hat{\nu}\pi\nu\kappa\nuo\hat{\nu}\mu\epsilon\tauo\chi\eta\nu$:¹⁵ that is, in effect, standing in the same functional role in a tetrachord. As represented in the $\kappa\alpha\tau\alpha\pi\dot{\nu}\kappa\nu\omega\sigma is$ diagram, *none* of the modes stands in this relation to any other, since, for arithmetical reasons which I shall pass over,¹⁶ such relations are possible only between modes whose $\mu\epsilon\sigma\alpha i$ are a tone, a fourth, a fifth or five tones apart, and no pairs of modes as represented in the diagram fulfil any of these conditions. $\kappa\alpha\tau\alpha\pi\dot{\nu}\kappa\nu\omega\sigma is$ is therefore useless as an attempt to explain intermodulation.

It is characteristic of Aristoxenus that although his comments could be extended to generate this result, he uses a more limited argument, and one designed to express something of the basis of these rules in sense perception. The $\delta\rho\mu\sigma\nu\kappa\sigma\ell$, he says (53), apparently discount ($\partial\lambda\nu\rho\rho\epsilon$) the proper ordering of melody, as is made clear $\epsilon\kappa$ $\tau\sigma\vartheta$ $\pi\lambda\eta\theta\sigma\nus$ $\tau\omega\nu$ $\epsilon\xi\eta s$ $\tau\iota\theta\epsilon\mu\epsilon\nu\omega\nu$ $\delta\iota\epsilon\sigma\epsilon\omega\nu$. For the voice cannot connect even as many as three dieses. This claim is elaborated in the alternate passage, 28. The voice, he says here, $\tau\eta\nu$ $\tau\rho\ell\tau\eta\nu$ $\delta\epsilon\epsilon\sigma\iota\nu$ $\pi\alpha\nua$ $\pi\sigma\iota\sigma\vartheta\sigma$ $\sigma\vartheta\chi$ $\sigma\iotaa$ $\tau\epsilon$ $\epsilon\sigma\sigma\iota$ $\pi\rho\sigma\sigma\tau\ell\theta\epsilon\nu\alpha\iota$, but if ascending after two dieses $\epsilon\lambda\alpha\chi\iota\sigma\tau\sigma\nu$ $\mu\epsilon\lambda\omega\delta\epsilon\iota$ $\tau\delta$ $\lambda\sigma\iota\pi\sigma\nu$ $\tau\sigma\vartheta$ $\delta\iotaa$ $\tau\epsilon\sigma\sigma\alpha\rho\omega\nu$ (the remainder of a fourth), and if descending $\tau\sigma\nu\iota\alpha\ell\sigma\nu$ $\epsilon\lambda\alpha\tau\tau\sigma\nu$ $\sigma\vartheta$ $\delta\nu\nu\alpha\tau\alpha\iota$ $\mu\epsilon\lambda\omega\delta\epsilon\iota\nu$. Any smaller movements are impossible. Hence, the moral is, one cannot reach the $\mu\epsilon\sigma\eta$ of the next key, as here represented, since it stands in a musically impossible relation to elements in the existing mode.

Now taken at face value this is both false and pointless. It is admittedly difficult to sing three quarter-tones in a row with any accuracy, but it is not impossible: even if it were, the thing can readily be done on a stringed instrument: and even if that were not so, the next possible upwards interval is certainly much less than the remainder of a fourth, which is two whole tones. Further, Aristoxenus himself has a long and bad-tempered passage explicitly aimed at refuting those who would base claims about music on the features and limitations of instruments (41-3). And again, merely to show that one cannot sing the continuous succession of intervals from one $\mu \acute{e}\sigma\eta$ to the next plainly fails to show that one cannot get there in practice by *any* means: one can after all readily skip a note and get there by the progression of a quarter-tone and a semitone.

Aristoxenus is not, I think, quite so stupid. His point is rather that to move to a position three dieses away from a pitch on our original scale, and already preceded in the structure of that scale by two shifts of a diesis each, is to move to a position which musically speaking does not exist. It is of the nature of melody $(\dot{\eta} \tau \eta s \mu \epsilon \lambda \omega \delta i a s \phi i \sigma \iota s,$ much in evidence in this passage) for the notes of a scale to be defined by their $\delta i \nu a \mu \iota s \sigma \eta$ musical function: when we move up by quarter-tones in the enharmonic scale from $\dot{\nu} \pi \dot{\alpha} \tau \eta$ to $\pi a \rho \upsilon \pi \dot{\alpha} \tau \eta$ to $\lambda \iota \chi a \nu \dot{\sigma} s$, there remains no functional location hence no note—short of $\mu \dot{\epsilon} \sigma \eta$, which invariably stands at a distance of a fourth from $\dot{\upsilon} \pi \dot{\alpha} \tau \eta$. These functions exist as natural and essential constituents of properly constructed melody, and the criterion of this, of the *identity* of this or that note as having a given $\delta i \nu a \mu \iota s$, rests with $a \ddot{\iota} \sigma \theta \eta \sigma \iota s$ coupled with $\mu \nu \eta \mu \eta$ and $\delta \iota a \nu o i a$. We might say that a sequence of notes which actually progressed, mathematically speaking, into this 'impossible' position would be heard either as not melodic at all $(\epsilon \kappa \mu \epsilon \lambda \epsilon s)$ or, perhaps, as involving a poor attempt to repeat the $\lambda \iota \chi a \nu \delta s$ or to reach the $\mu \epsilon \sigma \eta$.

We may well ask how Aristoxenus can be so sure of all this. It is not simply that he is committed to recognising the existing $\gamma \epsilon \nu \eta$ as representing the only possible forms of musical sequence, and the accepted $\delta \nu \nu \dot{a} \mu \epsilon \iota s$ as unique—these 'truths' come to him from aesthetic experience, and cannot, perhaps, be judged further. But his arguments also depend on the attribution to the given $\gamma \epsilon \nu \eta$ of a mathematically expressed intervallic structure, and on the possibility of pinning down quantitatively—even if, admittedly, over a $\tau \delta \pi \sigma s$ —the intervals between notes of given $\delta \nu \nu \dot{a} \mu \epsilon \iota s$. This obviously is not given in the direct perception of an interval, taken by itself. We need a $\mu \epsilon \tau \rho \sigma \nu$, a standard of measurement, to which we can refer the heard intervals, and on Aristoxenean principles it must be one specifiable in relation to some identifiable object of $a \tilde{a} \sigma \theta \eta \sigma \iota s$, not merely e.g. a mathematical ratio.

For Aristotle (cf. Met. I 1053a10 ff., 1053b32 ff., N 1087b33) the musical $\mu \epsilon \tau \rho \sigma \nu$ is the smallest musical interval, the diesis or quarter-tone. For Aristoxenus too the smallest $\mu \epsilon \lambda \omega \delta \sigma \delta \mu \epsilon \nu \sigma \nu$ is the quarter-tone. But in his scheme of things we cannot use it as a $\mu \epsilon \tau \rho \sigma \nu$ or a starting point ($d\rho \chi \eta$) for definition. Each of the first principles of the science must be $\tau \sigma \sigma \sigma \sigma \sigma \sigma \delta \sigma \delta \sigma \tau \sigma \sigma \tau \eta s$ alogh $\eta \sigma \epsilon \omega s$ $\sigma \nu \sigma \rho \delta \sigma \delta \sigma \tau \tau \sigma \nu \tau \eta s$ $\delta \sigma \mu \sigma \nu \kappa \eta s$ $\pi \rho \alpha \mu \mu \sigma \epsilon \epsilon \delta s$, recognisable as a principle by $a \delta \sigma \eta \sigma \sigma s$: and if we fail to fulfil this condition we shall find ourselves falling $\epsilon s \tau \eta \nu \delta \sigma \epsilon \rho \rho \delta \sigma \nu$, by beginning from facts or assumptions extraneous to the nature of sound as heard (44). And to take the enharmonic diesis as an $d\rho \chi \eta$ would not fulfil this requirement. Aristoxenus explains why in section 55.

Given some pitch as starting point, a particular quality of *discord* constructed on it will be producible not just by a note at some unique locus, but by one at any locus over a range ($\tau \delta \pi \sigma s$). Within that range there is no distinction of heard quality in the discord; hence, from a musical point of view, it is the same discord, and it would be a mistake comparable to those discussed earlier even to try to pin it down 'accurately' to a particular size of interval. There is no such thing as this 'accuracy'. It follows that no discord will do to establish a heard point of reference to which we may relate the sizes of other intervals: and the dises is of course a discord.

So Aristoxenus turns to concords, which, so he claims, are definitely determined to a particular magnitude— $\delta\lambda\omega s \ o\lambda\kappa \ excut \tau \delta\pi ov \ d\lambda\lambda' \ ev \ \mu ev \ ed e \ u \ box{i} \ od e \ u$. We can identify the fourth, the fifth and the octave definitely and precisely by ear. Effectively, though for most purposes Aristoxenus's explicit $\mu \ ev \ ev \ v$ is the tone, a discord, the reference point for all measurement of intervals is a concord, or rather the first two concords taken as a pair. As we saw earlier, the tone is defined, stipulatively but on the basis of existing tradition, as the difference between a fourth and a fifth.

Given that, it is possible to ascertain the sizes of other intervals relatively to the tone by an ingenious method of construction involving concords only, and thus capable of being checked against the evidence of $aio \theta\eta\sigma is$ (55–7). Thus, in practical musicianship, if for example we want to find a note two tones below a given note, we do so by finding the fourth above, the fifth below that, the fourth above that again, and finally the fifth below that (55). More importantly for the purposes of musical theory, we can demonstrate by the same method that, for instance, the fourth itself is an interval of $2\frac{1}{2}$ tones, and can use this (actually highly controversial) putative fact in subsequent arithmetical analysis.¹⁷ (The first example will play its part in theory too, since it is required for the demonstration of the size of the fourth; see sections 56–7.)

Aristoxenus obviously considers all this crucially important, and fundamental to the arithmetical conclusions which he draws. There is no way conformable to his views about the primacy of $ai\sigma\theta\eta\sigma s$, other than by this 'principle of concordance', that we can accurately establish the size of an interval in relation to the tone: and given this principle it is possible to use abstract arithmetical reasoning concerning the relations between the intervals so specified.

But it does not seem to be enough for his purposes. The point I wish to make is this. The principle of concordance, useful though it is, will not allow us to construct intervals smaller than the semitone. (Semitones are constructed quite legitimately in the demonstration of the size of the fourth.) Of course we can if we wish for the purposes of theoretical analysis *talk* about intervals

¹⁷ Cf. Euclid Sect. Can. 15.

smaller than that: Aristoxenus in his calculations mentions intervals as small as one twelfth of a tone, far smaller even than the least $\mu\epsilon\lambda\phi\deltao'\mu\epsilon\nuo\nu$. But what we cannot do is to establish by reference to perception that this or that heard interval *is* one third or one quarter of a tone. Thus, it appears, it is no good Aristoxenus asserting that the quarter-tone is the least $\mu\epsilon\lambda\phi\deltao'\mu\epsilon\nuo\nu$, and is the interval between this and that note of the enharmonic scale: for what counts as the least $\mu\epsilon\lambda\phi\deltao'\mu\epsilon\nuo\nu$ and what counts as being the enharmonic scale are, on his own principles, determined directly by $ai\sigma\theta\eta\sigma\iotas$, by ear, not by any abstract mathematical considerations. There simply is no way of showing that this interval, heard as the space between enharmonic $\dot{\nu}\pi \dot{\alpha}\tau\eta$ and $\pi a\rho\nu\pi\dot{\alpha}\tau\eta$, stands in just that mathematical relation to the tone. Of course, Aristoxenus may in part be recognising this when he grants range, $\tau \dot{\sigma}\pi\sigma s$, not absolute location, to certain of the notes bounding these intervals: but his desire for systematisation outstrips his equipment even so, since he insists on giving arithmetical values to the extent of these $\tau \dot{\sigma}\pi\sigma\iota$, values which still require us to recognise the precise interval of a quarter-tone. And if he is not allowed this degree of precision, a great deal of the detailed derivation of theorems in Book III must be without foundation.

Aristoxenus was an innovator, consciously and often bumptiously so. His objective was to claw back the study of music from the hands of physicists, mathematicians, and mere recorders of low-level empirical fact, and to establish it as an independent science having its own laws and principles, and a subject matter with its own distinctive $\phi \dot{\upsilon} \sigma_{i\sigma}$. Problems arising from the facts of musical experience—why this is a possible melody while that is not, why some modulations are possible and not others, in what relations the heard intervals stand to one another, in what the identity of notes in a scale consists, and so on-all these are to be explained not in terms of the physics of sound production or by abstract mathematical considerations, but through principles inherent in our experience of sound as musical, and depending ultimately on $ai\sigma\theta\eta\sigma_{i\sigma}$, on what we perceive as melodious, concordant, and the like. His contribution to the study of music is significant, and goes far beyond anything I have said in this paper: and so, I think, is his contribution to our understanding of the notion of an independent science in the Aristotelian mould. But I have argued that in crucial respects he mistook the proper direction of his science, and overstepped the limits which his methodological principles laid down. Perhaps the influence of his reputedly Pythagorean upbringing, though he explicitly rejected all that it stood for, made the Siren-song of Number in the end too seductive.

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